Energy-Efficient Multi-Channel Cooperative Sensing Scheduling with Heterogeneous Channel Conditions for Cognitive Radio Networks

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Abstract-Spectrum sensing is an important aspect of Cognitive Radio Networks. Secondary users should sense the channels periodically in order to ensure primary user protection. Sensing with cooperation among several secondary users is more robust and less error prone. However, cooperation also increases the energy spent for sensing. Considering the periodic nature of sensing, even a small amount of savings in each sensing period leads to considerable improvement in the long run. In this paper, we consider the problem of energy-efficient spectrum sensing scheduling with satisfactory primary user protection. Our model exploits the diversity of secondary users in their received signal-to-noise-ratio value of the primary signal to determine the sensing duration for each user/channel pair for higher energy efficiency. We model the mentioned problem as an optimization problem with two different objectives. The first one minimizes the energy consumption whereas the second one minimizes the spectrum sensing duration in order to maximize the remaining time for data transmission. We solve both problems using outer linearization method. In addition, we present two sub-optimal but efficient heuristic methods. We provide an extensive performance analysis of our proposed methods under various number of secondary users, average channel signal-tonoise-ratio, and channel sampling frequency. Our analysis reveal that all proposals with energy minimization perspective provide significant energy savings compared to a pure transmission time maximization technique.

Index Terms—Cooperative sensing scheduling, energy-efficient sensing, sensing task assignment, heterogeneous sensing.

I. INTRODUCTION

I NCREASING demand for wireless communications calls for better spectrum utilization. One of the most promising solutions is the dynamic spectrum access (DSA) paradigm which allows the opportunistic access of spatio-temporally unused wireless spectrum by cognitive radio networks (CRN). In a CRN, a secondary user (SU) transmits through a frequency channel which is licensed to primary users (PU) but is currently unoccupied. However, DSA has the challenge to guarantee that SUs vacate the spectrum whenever a PU transmits simultaneously at the band of SU transmission. Hence, SUs must perform spectrum sensing. The accuracy of spectrum sensing is paramount for both finding the spectral

Copyright (c) 2013 IEEE. Personal use of this material is permitted. However, permission to use this material for any other purposes must be obtained from the IEEE by sending a request to pubs-permissions@ieee.org. Salim Eryigit and Tuna Tugcu are with the Department of Computer Engineering, Bogazici University, Istanbul, 34342 Turkey, e-mail: {eryigit, tugcu@boun.edu.tr}. Suzan Bayhan is with Helsinki Institute for Information Technology HIIT, Aalto University, Finland, e-mail: {bayhan@hiit.fi}. voids and for protecting the PU communications. Hence, a sensing period is reserved at the beginning of each frame for the spectrum sensing task.

Previous works showed the increase in sensing accuracy with the increase in sensing time [1]. On the other hand, SUs which are mostly mobile devices should be energy-efficient as they use their battery power. Therefore, from the energy (throughput) efficiency perspective, the more time is spent on sensing the more energy is consumed for overhead and less time remains for transmission. On the other hand, the throughput of the network is a function of the detection accuracy (i.e., probability of detection, P_d , and probability of false alarm, P_f). Hence, there is a trade-off between sensing and transmission durations for both throughput and energy efficiency.

In addition, it is shown that cooperation among the SUs increases the detection reliability of spectrum sensing at the expense of additional communication overhead that increases with the number of cooperating SUs [2]. Different from cooperative sensing in a single channel, *cooperative sensing scheduling* (CSS) has to balance the trade-off between the detection accuracy of a single channel and the number of channels being sensed in a multi-channel CRN. That is to say, *the more SUs are assigned to sense a single channel, the higher is the probability of detection for that channel at the expense of leaving some channels being unexplored.* While cooperative sensing has been well-investigated, CSS still remains unexplored. It is shown in previous works that CSS is NP-hard [3]. Taking the energy efficiency concerns into account makes this problem even more complicated.

In this paper, we focus on energy efficiency of cooperative spectrum sensing in a multi-channel CRN with heterogeneous PU channels in terms of received signal-to-noise ratio (SNR) values. Since scheduling the SUs to sense a number of channels in a CRN is a difficult task [3], we propose three schemes for energy-efficient CSS. The first one uses outer linearization to find the optimal solution whereas the latter two are efficient heuristic methods. Apart from these three, we also analyze the problem from the transmission time point of view as time spent for sensing is also time lost for transmission.

The rest of the paper is organized as follows: In Section II, we revise the related work on energy efficiency in spectrum sensing and state our contributions to the literature. In Section III, we define the cooperative sensing system model and introduce the basic theorems used in the formulated energyefficient CSS scheme. Section IV first formulates the problem and presents the methodology for finding the optimal solution. Proposed heuristic schemes are described in Section V and their performances are evaluated in Section VI. Finally, Section VII concludes the paper.

II. RELATED WORK AND CONTRIBUTIONS

Energy efficiency of CRNs has recently gained interest and most of the initial works focus on the energy efficiency of spectrum sensing [4–9] and cooperative sensing [10–14]. Su et al. minimize the sensing energy consumption while meeting the constraint on undiscovered spectrum opportunities [5], and adapt the period of spectrum sensing in order to attain a balance between energy consumption and missed spectrum opportunities for a random access CRN [7]. Optimal sensing and transmission durations for an SU under both high and low SU power capacity are analytically derived in [8]. The effect of transmission, idling, and sensing power consumption are analyzed in that work. Pei et al. devise an optimal policy for a single SU to decide on the order of channels to be sensed as well as when to stop sensing and start transmission [9]. While all these previous works have valuable contributions, they fall short of practicality. In practical CRNs, there are multiple and most likely heteregenous primary channels. In this setting, one of the major concern of the operator is to explore as many PU channels as possible meeting the PU detection and false alarm constraints. Therefore, in our work, we enforce the SUs to sense all PU channels colloboratively to maximize the discovered spectrum opportunities.

Works in [12] and [13] consider the communication cost for determining the number of cooperating SUs. Maleki et al. find the minimum number of cooperating SUs that attains the required detection and false alarm probability performance [12]. The fewer SUs are engaged in sensing, the less time is spent for reporting the sensing outcomes, and thereby the more time is spent for transmission. Moreover, sensing reports from unreliable SUs may decrease the sensing performance. SUs with unreliable sensing information are refrained from reporting their sensing results to save energy in [13]. In addition, a cluster-based decision collection instead of a high power-consuming broadcasting scheme, is also proposed in [13]. The cluster-based scheme adapts the transmission power considering the most distant node. The decision fusion rule (i.e., how the collected sensing information is processed to give the final decision on the existence of PU) at the fusion centre also affects the energy efficiency. Peh et al. [14] tune the k parameter in k-out-of-N fusion rule at each frame as well as the threshold for energy detection scheme at the fusion centre. Heterogeneity of PU channels and SU link conditions were ignored in these works making the investigated scenarios less realistic. In addition, assigning the same sensing duration for all SUs regardless of their link conditions may result in waste of energy at SUs with good link conditions. In contrast to these works, we incorporate the effect of signal-to-noise ratios of SUs into our scheme to calculate appropriate sensing duration for each SU and frequency pair.

Works in [15, 16] and [17] present various solutions for improving the energy-efficiency of CSS. Sensing scheduling is 2

modelled as a utility maximization problem subject to a certain cooperative detection probability in [15]. In addition, a constraint on minimum discovered transmission time is imposed in order to ensure a certain QoS together with heterogeneous detection probability requirements. Similarly, Zhang et al. determine the number of SUs to sense each channel as well as the sensing duration in a slot [16] while Hao et al. study the optimal partition of the SUs into coalitions such that the total energy efficiency of all coalitions are maximized [17]. Work in [16] utilizes partially observable Markovian decision process (POMDP) framework and tunes the punishment parameter for higher energy efficiency. A distributed solution using coalition formation is proposed in [17].

Apart from these works, the main contributions of this paper can be summarized as follows:

- We consider a scenario in which the number of SUs is larger than the number of primary channels. Therefore, our main concern is to select the SUs to sense all channels while the works in [15, 16] and [17] select a subset of primary channels to be sensed by all SUs. In addition, in the previous works an SU can sense at most one channel whereas in this work, SUs can sense multiple channels as long as they finish sensing in the dedicated time.
- Unlike these works, we account for the heterogeneity of the SU link conditions (i.e., received SNR of the PU signal at the SU). Therefore, our CSS solution additionally determines which SUs should sense a channel. Our paper diverges from the previous works, which only determine the number of SUs to sense a specific primary channel.
- Moreover, sensing duration associated with an SU is adjusted according to the link SNR as opposed to the prior works, which consider identical sensing duration for all SUs. Simply, our approach bases on the fact that channels with high SNR values require shorter sensing time for a required detection probability and false alarm probability. Hence, an SU can save energy by sensing one of the channels with higher SNR as opposed to the fixed sensing duration scheme.

III. SYSTEM MODEL

We assume an infrastructure based CRN with N secondary users, M channels, and a Cognitive Radio base station (CBS). Our consideration is a specific case where the number of channels is less than the number of SUs, i.e. $N \gg M$. We believe that in a cellular network this assumption generally holds as there are lots of users within the coverage area of the base station. If that is not the case, the CBS may select a subset of the channels based on their past data like availability, capacity, etc. such that there are enough SUs to sense all selected channels. This selection procedure has the potential to reduce energy consumption by eliminating the less favorable channels. SUs operate in a time synchronized manner within a frame based communication protocol. Each frame starts with a fixed length quiet sensing period of duration T^s during which SUs sense the channel(s) assigned to them. An SU may sense multiple channels during the quiet period as long as the total time dedicated to sensing by the SU does not exceed T^s .

Then, all SUs that sense at least one channel report their hard decisions about these channels (0 or 1, indicating the absence or presence of primary user) to the CBS. We assume that the secondary network has a dedicated common control channel that is used for this reporting task and other control messages. The CBS combines the decisions using OR rule. The remaining time is used for transmission. We also assume that the SUs and the channels are heterogeneous. That is to say, the SNR of each SU over each channel is different due to different proximities from the PUs and different channel conditions (shadowing, fading, etc.). We assume the existence of a receiver block at each SU to estimate the SNR level and feedback it to the CBS through the error-free feedback channel [18]. With the help of the receiver block, we assume that the instantaneous SNR values are known. However, if that is not the case, long term SNR values can also be used. This time, the techniques discussed in this paper can also be applied. However, the main objective becomes the minimization of expected energy, instead of the actual one. The frame structure is shown in Fig. 1 where T, T^{rep} , and $\tau_{m,n}$ are the total frame length, the time dedicated for reporting sensing results, and the time that SU_n senses channel m, respectively.



Fig. 1: A frame starts with a sensing period followed by reporting and transmission periods.

Our main goal is to sense all M channels with minimum energy and sufficient accuracy such that cooperative detection probability of each channel is greater than some predefined threshold value (denoted by ${}_{th}Q^d$) and cooperative false alarm probability is smaller than another threshold (denoted by ${}_{th}Q^f$). Since channel sensing consumes energy, an SU may not utilize all of the quiet period duration for sensing if not necessary. On the other hand, it is desirable to sense a channel with a couple of SUs instead of a single SU (even though, it may satisfy the thresholds) in order to increase robustness. Hence, there is a trade-off between energy consumption and sensing reliability. The problem includes the assignment of SUs to channel(s) for the sensing task together with the decision of the sensing time for the channel(s) to be sensed by each SU.

Let $P_{m,n}^f$, $P_{m,n}^d$, $\gamma_{m,n}$ denote the probability of false alarm, probability of detection, SNR for SU_n over channel m, respectively. If we assume that $P_{m,n}^f$ is fixed, then for a complex-valued PSK channel with circularly symmetric complex Gaussian noise, $P_{m,n}^d$ is given by [1]

$$P_{m,n}^d = \mathcal{Q}\left(\frac{\mathcal{Q}^{-1}(P_{m,n}^f) - \sqrt{\tau_{m,n} f_s} \gamma_{m,n}}{\sqrt{2\gamma_{m,n} + 1}}\right)$$
(1)

where f_s is the sampling frequency and Q is the complementary cumulative distribution of a standard Gaussian.

Theorem 1: $P_{m,n}^d$ is an increasing function of $\tau_{m,n}$. Furthermore, it is also concave if

$$-\frac{1}{\sqrt{\tau_{m,n}}} + \frac{\gamma_{m,n}\sqrt{f_s}}{(2\gamma_{m,n}+1)} (\mathcal{Q}^{-1}(P^f_{m,n}) - \sqrt{\tau_{m,n}f_s}\gamma_{m,n}) < 0.$$
(2)

This theorem is well known, and its proof is given in Appendix A.

Lemma 1: $P_{m,n}^d$ is a concave function of $\tau_{m,n}$ if $P_{m,n}^d > 0.5$.

Lemma 1 is a straightforward application of Theorem 1. The proof can be found in Appendix B.

Lemma 2: $1 - P_{m,n}^d$ is a non-negative decreasing function of $\tau_{m,n}$. It is also convex if the condition in (2) is satisfied.

Let S_m and Q_m^d denote the set of SUs sensing channel m, and cooperative detection probability for channel m, respectively. Using OR rule for decision combining gives

$$Q_m^d = 1 - \prod_{n \in \mathcal{S}_m} (1 - P_{m,n}^d)$$
$$= 1 - \prod_{n \in \mathcal{S}_m} \left(1 - \mathcal{Q} \left(\frac{\mathcal{Q}^{-1}(P_{m,n}^f) - \sqrt{\tau_{m,n} f_s} \gamma_{m,n}}{\sqrt{2\gamma_{m,n} + 1}} \right) \right)$$

Theorem 2: Q_m^d is an increasing function of $\tau_{m,n}$. Moreover, it is also concave if the condition in (2) is satisfied $\forall n \in S_m$.

The proof of this theorem is given in Appendix C.

IV. Optimization Model and Solution Methodology

A. Energy Consumption Model

Let P^s and $E^s_{m,n}$ be the power consumed during channel sensing and energy dissipated by SU_n for sensing channel m, respectively. $E^s_{m,n}$ is equal to $P^s\tau_{m,n}$. Then, energy consumption for channel sensing (denoted by E^s) can be written as

$$E^s = \sum_{m=1}^M \sum_{n=1}^N P^s \tau_{m,n}$$

Besides channel sensing, SUs also consume energy by transmitting their local results to the CBS. We assume that SU transmits its sensing report as a single packet regardless of the number of channels sensed, and the reporting period is long enough such that all SUs can send their packets. Let E_n^{rep} denote the energy consumed for reporting the sensing result to CBS, which depends on the location of SU_n relative to the CBS. In addition, let S^{rep} denote the set of SUs that perform sensing in this frame that are required to report their local decisions to the CBS. Then, the total energy consumption for reporting is given by

$$E^{rep} = \sum_{n \in \mathcal{S}^{rep}} E_n^{rep}.$$

This model assumes that all reporting packets are transmitted successfully. If that is not the case, the model can be modified as follows: Let p denote the probability of successful packet transmission that is geometrically distributed, then the expected number of transmission attempts for an SU is given by 1/p, and E^{rep} is given by $E^{rep} = 1/p \sum_{n \in S^{rep}} E_n^{rep}$.

B. Optimization Model for Energy Efficient (EE) Sensing

We first define the decision variables that are used in the optimization model. Let

$$\tau_{m,n} = \text{time spent by } SU_n \text{ for sensing channel } m,$$

$$x_{m,n} = \begin{cases} 1, \text{ if channel } m \text{ is sensed by } SU_n \\ 0, \text{ o/w.} \end{cases},$$

$$y_n = \begin{cases} 1, \text{ if } SU_n \text{ transmits sensing result to CBS} \\ 0, \text{ o/w.} \end{cases}$$

From (1), for a given $P_{m,n}^d$ value the required $\tau_{m,n}$ can be written as

$$\tau_{m,n} = \left(\frac{\mathcal{Q}^{-1}(P_{m,n}^f) - \mathcal{Q}^{-1}(P_{m,n}^d)\sqrt{2\gamma_{m,n} + 1}}{\gamma_{m,n}\sqrt{f_s}}\right)^2.$$
 (3)

In addition, let $\tau_{m,n}^{min}$ denote the sensing time required for SU_n in order to achieve a $P_{m,n}^d$ value of 0.5. It can be calculated from (3) as

$$\tau_{m,n}^{min} = \left(\frac{\mathcal{Q}^{-1}(P_{m,n}^f)}{\gamma_{m,n}\sqrt{f_s}}\right)^2.$$

We assume that a channel should be sensed by at least δ^{min} SUs. δ^{min} defines the minimum number of cooperating SUs for a channel. The selection of δ^{min} value is a design criterion. In order to encourage cooperation and improve robustness, a δ^{min} value greater than one is preferred. On the other hand, regarding energy efficiency concern, δ^{min} should not be high as each additional SU used for sensing incurs sensing energy consumption, and maybe reporting energy.

If we assume that $P_{m,n}^f = P^f \ \forall m, n$, then Q_m^f is given by

$$Q_m^f = 1 - \prod_{n \in \mathcal{S}_m} (1 - P^f).$$

Since $Q_m^f \leq {}_{th}Q^f$, then the maximum number of cooperating SUs, denoted by δ^{max} , can be calculated as

$$\delta^{max} = \lfloor \frac{\log\left(1 - {}_{th}Q^f\right)}{\log\left(1 - P^f\right)} \rfloor.$$
(4)

In other words, δ^{max} is the maximum number of cooperating SUs that satisfy the cooperative false alarm constraint. The solution methodology we apply can also be used for the case where $P_{m,n}^f$ values differ. We discuss this case in detail at the end of the following section. The optimization model can be written as

P1: min
$$w = \sum_{m=1}^{M} \sum_{n=1}^{N} P^s \tau_{m,n} + \sum_{n=1}^{N} E_n^{rep} y_n$$
 (5)

subject to:

$$\tau_{m,n} \ge \tau_{m,n}^{\min} x_{m,n} \qquad \forall m \in M, \forall n \in N$$
(6)

$$\sum_{m=1}^{M} \tau_{m,n} \le T^{s} y_{n} \qquad \forall n \in N$$
(7)

$$\sum_{n=1}^{N} x_{m,n} \ge \delta^{min} \qquad \forall m \in M \tag{8}$$

$$\sum_{n=1}^{N} x_{m,n} \le \delta^{max} \qquad \forall m \in M \tag{9}$$

$$\sum_{m=1}^{M} x_{m,n} \le M y_n \qquad \forall n \in N \tag{10}$$

$${}_{th}Q^d - Q^d_m \le 0 \qquad \forall m \in M \tag{11}$$

$$x_{m,n}, y_n \in \{0, 1\} \qquad \forall m \in M, \forall n \in N$$
(12)

$$\tau_{m,n} \ge 0 \qquad \forall m \in M, \forall n \in N,$$
 (13)

where this time Q_m^d is defined as

$$Q_m^d = 1 - \prod_{n=1}^N \left(1 - \mathcal{Q}\left(\frac{\mathcal{Q}^{-1}(P^f) - \sqrt{\tau_{m,n} f_s} \gamma_{m,n}}{\sqrt{2\gamma_{m,n} + 1}}\right) x_{m,n} \right)$$

Hence, SUs with $x_{m,n}$ value of 0 contribute 1 to the above multiplication, whereas those with $x_{m,n}$ value of 1 contribute $(1 - P_{m,n}^d)$.

The objective in (5) minimizes the total energy consumption associated with sensing for a frame. Constraint (6) specifies that if SU_n senses channel m, the sensing duration should be at least $\tau_{m,n}^{min}$. In this way, we guarantee that the concavity condition always holds. Constraint (7) denotes that total time spent by an SU for sensing should be less than or equal to the sensing duration of a frame. It also forces all $\tau_{m,n}$ values associated with SU_n to zero, if $y_n = 0$. Constraint (8) requires that each channel should be sensed by at least δ^{min} SUs. Similarly, Constraint (9) limits the number of cooperating SUs for a channel in order to satisfy the false alarm probability threshold. Constraint (10) forces y_n value for an SU to 1, if that SU senses any channels. The requirement for cooperative detection probability being greater than the threshold for each channel is expressed by Constraint (11). Finally, Constraints (12) and (13) specify the types of variables.

The above problem is a Mixed Integer Non-linear Programming problem because of Constraint (11), even though its objective is linear. We resort to the outer linearization algorithm to solve the above problem.

C. Outer Linearization

As proven before, once the $x_{m,n}$ values are fixed, Q_m^d value is concave in terms of $\tau_{m,n}$. Thus, Constraint (11) is convex, and the outer linearization procedure can be used to find the optimal solution [19]. Outer linearization works by first ignoring the mixed integer non-linear constraints to obtain an initial solution. If the solution satisfies all previously ignored constraints, then it is optimal. On the other hand, if it does not, then the most violated constraint is linearized using the current solution, and added to the current problem as a new constraint to obtain another solution. The linearization process goes on until all constraints are satisfied with an ϵ tolerance. Since the constraints are convex, the procedure is guaranteed to terminate in finite number of steps [20]. The steps of the procedure are as follows:

- Step 1: Initialize the iteration counter, k = 1. Solve the initial Mixed Integer Linear Programming problem (P2) formed by ignoring Constraint (11), and obtain the initial solution $\tau_{m.n}^1$, $x_{m.n}^1$, y_n^1 .
- Step 2: Identify the most violated constraint, g_m , among the M constraints of (11) with the current solution ($\tau_{m,n}^k$, $x_{m,n}^k$, and y_n^k). That is to say, g_m is the cooperative detection probability constraint corresponding to the channel that deviates from the threshold value most. Let v_m denote the corresponding deviation.
- Step 3: If the maximum violation is smaller than *ε*, stop; the current solution is optimal with *ε* feasibility tolerance. Otherwise, proceed with Step 4.
- Step 4: Linearize the most violated constraint by adding the following linear constraint to **P2**:

$$\nabla g_m(\dots x_{m,i}^k, \dots \tau_{m,i}^k, \dots)^T \begin{pmatrix} \vdots \\ x_{m,i} - x_{m,i}^k \\ \vdots \\ \tau_{m,i} - \tau_{m,i}^k \\ \vdots \end{pmatrix} + v_m \le 0$$

where $\nabla g_m(\ldots x_{m,i}^k, \ldots \tau_{m,i}^k, \ldots)$ is the gradient of g_m evaluated at the current solution. Its individual entries are given by

$$\frac{\partial g_m}{\partial x_{m,i}} = -\mathcal{Q}\left(\frac{\mathcal{Q}^{-1}(P^f) - \sqrt{\tau_{m,i}f_s}\gamma_{m,i}}{\sqrt{2\gamma_{m,i}+1}}\right) B_{m,i}$$

$$\frac{\partial g_m}{\partial \tau_{m,i}} = -\frac{x_{m,i}\gamma_{m,i}\sqrt{f_s}}{2\sqrt{\tau_{m,i}}\sqrt{2\pi}\sqrt{2\gamma_{m,i}+1}} A_{m,i}B_{m,i}$$

where $B_{m,i}$ is given by

$$\prod_{n=1,n\neq i}^{N} \left[1 - \mathcal{Q}\left(\frac{\mathcal{Q}^{-1}(P^f) - \sqrt{\tau_{m,n} f_s} \gamma_{m,n}}{\sqrt{2\gamma_{m,n} + 1}} \right) x_{m,n} \right) \right].$$

Set k = k + 1, solve the current problem to obtain $\tau_{m,n}^k$, $x_{m,n}^k$, and y_n^k values. Proceed with Step 2.

In the remainder of this paper, we refer to the application of outer linearization to Problem P1 as EE, which stands for energy efficiency.

For the case where $P_{m,n}^f$ values differ, false alarm constraint assumes the following form

$$1 - \prod_{n=1}^{N} (1 - P_{m,n}^{f} x_{m,n}) - {}_{th} Q^{f} \le 0.$$

The outer linearization procedure can still be applied in this case, but this time 2M constraints (cooperative false alarm probability constraint in addition to cooperative detection probability constraint for each channel) need to be checked for feasibility. The other steps of the procedure are the same.

D. Transmission Time Maximization (TXT)

The aforementioned model optimizes the total energy dedicated to the sensing task while achieving satisfactory sensing

performance in terms of detection and false alarm probabilities. However, in this approach, sensing duration of a frame (denoted by T^s) is constant. Hence, if we denote the frame duration by T and reporting time of the sensing outcomes by T^{rep} , which are also constant, then the transmission time for data packets is given by $T - T^s - T^{rep}$. Another approach is to maximize the data transmission duration of a frame. This time, we treat T^s as a decision variable. Assuming a quiet sensing period, T^s is given by $\max_n \{\sum_{m=1}^M \tau_{m,n}\}$. In other words, T^s is the maximum of total sensing times for all SUs as the network should wait for the SU with the longest total sensing time before moving on the next phase of a frame. Then, the objective becomes max $z_1 = T - T^{rep} - \max_n \{\sum_{m=1}^M \tau_{m,n}\}$. Since T and T^{rep} are constants, this objective is equivalent to min $z_2 = \max_{n} \{ \sum_{n=1}^{M} \tau_{m,n} \}$ subject to Constraints (6), (8), (9), (10), (11), (12), and (13). To solve this problem, we resort to the outer linearization procedure again as the constraints are almost the same.

V. HEURISTIC APPROACHES

In this section, we propose two suboptimal but fast heuristic approaches for the energy-efficient sensing problem. The first one focuses on greedily minimizing sensing energy while disregarding the reporting energy. On the other hand, the second heuristic initially considers the reporting energy, then it regards the sensing energy.

Unlike the previous two approaches that support different detection probabilities for different channel and user pairs, these heuristics require a fixed detection probability, P^d , for all channels and users for the sake of simplicity and quick execution time. This approach is frequently applied in the literature [4, 12, 14]. For both heuristics, we sense each channel with δ^{min} SUs. Thus, the required P^d value can be calculated as

$$P^{d} = \max\{1 - (1 - {}_{th}Q^{d})^{1/\delta^{min}}, P^{d}_{min}\}$$

which guarantees a minimum detection probability of P_{min}^d . As the $P_{m,n}^f$ values are assumed to be the same for all SUchannel pairs as before, the goal of the heuristics is to find the best SU/channel assignment.

A. Sensing Energy Minimization Heuristic (SEM)

This heuristic minimizes the sensing energy by selecting SUs with high SNR values for a channel while disregarding reporting energy. Initially, remaining sensing time of all SUs are equal to T^s . The heuristic starts with the first channel, sorts the SUs in descending order based on their $\gamma_{m,n}$ values, and selects the first SU in the list. Then, it calculates the required $\tau_{m,n}$ value for the selected SU to obtain a detection probability of P^d . If the remaining sensing time of the selected SU is greater than $\tau_{m,n}$, the selected SU is assigned to sense channel m. Otherwise, we move on to the next SU. The algorithm runs until δ^{min} SUs are assigned to all channels. The pseudo code for this heuristic is given in Algorithm 1.

Algorithm 1 Sensing Energy Minimization Heuristic

Require: P^d , δ^{min} , M, N, $\gamma_{m,n}$, T^s

1: $remainingTime[n] = T^s$ $\forall n$

2: for m = 1 to M do

- 3: Sort SUs in descending order of $\gamma_{m,n}$ and let *index* be the list of indices of the sorted entries such that *index*[1] corresponds to the index of SU with the highest $\gamma_{m,n}$ and *index*[N] corresponds to the index of SU with the lowest $\gamma_{m,n}$.
- 4: assignmentNo = 0, k = 1
- 5: while $assignment No < \delta^{min}$ do
- 6: n = index[k]

7: Select SU_n as a candidate and calculate $\tau_{m,n}$ value to achieve P^d using (3).

8: if
$$\tau_{m,n} \leq remainingTime[n]$$
 then

9:
$$remainingTime[n] = remainingTime[n] - \tau_{m,n}$$

10: $assignmentNo = assignmentNo + 1$
11: **end if**

 $12: \qquad k = k + 1$

- 13: end while
- 14: **end for**

Starting with the first channel, the heuristic selects δ^{min} SUs with the best $\gamma_{m,n}$ values and enough remaining sensing time for the sensing task. The outer loop takes O(M) steps. Sorting SUs based on their $\gamma_{m,n}$ values is $O(N \log N)$, whereas the inner loop is O(N). Hence, the total running time is $O(MN \log N)$.

B. Reporting Energy Minimization Heuristic (REM)

The main difference between the Reporting Energy Minimization (REM) heuristic and SEM is that REM first considers SUs that are already assigned to sense a channel. Let S^{rep} be the set of SUs that are going to perform sensing and transmit their reports for this frame. Similarly, S^{nrep} is the set of SUs that are not assigned to sense a channel yet. Initially, $S^{rep} = \emptyset$, $S^{nrep} = \{SU_1, SU_2, \ldots, SU_N\}$. The heuristic first looks for SUs among the ones in S^{rep} in order to save reporting energy. If enough SUs are not found, then it moves on to S^{nrep} . As in the previous case, SUs in S^{rep} and S^{nrep} are processed in decreasing order of $\gamma_{m,n}$ values for the considered channel. The pseudo code of REM is given in Algorithm 2.

This time both inner while loops (line 6 and line 18) take O(N), and the sorting operations are still $O(N \log N)$. As in the previous case, the total running time is $O(MN \log N)$.

VI. RESULTS

We assume that received SNR at an SU $(\gamma_{m,n})$ follows an exponential distribution with mean μ^{SNR} . In order to be consistent, we use the same $\gamma_{m,n}$ values for a given μ^{SNR} across different runs. For a given parameter set, we first run the TXT method to obtain the ideal sensing time denoted by T_{opt}^s . For the other methods, we scale this value with an α value ($\alpha > 1$), and use αT_{opt}^s as the sensing time for the

Algorithm 2 Reporting Energy Minimization Heuristic

Require: P^d , δ^{min} , M, N, $\gamma_{m,n}$, T^s 1: $remainingTime[n] = T^s \quad \forall n$ 2: $\mathcal{S}^{rep} = \emptyset, \, \mathcal{S}^{nrep} = \{SU_1, SU_2, \dots, SU_N\}$ 3: for m = 1 to M do Sort SUs in \mathcal{S}^{rep} in descending order of $\gamma_{m,n}$ and let 4. *indexRep* be the list of indices of the sorted entries. assignment No = 0, k = 15: while $(assignment No < \delta^{min}) \&\& (k \le |\mathcal{S}^{rep}|)$ do 6: n = indexRep[k]7: Select $SU_n \in S^{rep}$ as a candidate and calculate $\tau_{m,n}$ 8. value to achieve P^d using (3). if $\tau_{m,n} \leq remainingTime[n]$ then 9: remainingTime[n] = remainingTime[n] -10: $\tau_{m,n}$ assignmentNo = assignmentNo + 111: end if 12: k = k + 113: end while 14: if $assignment No < \delta^{min}$ then 15: Sort SUs in \mathcal{S}^{nrep} in descending order of $\gamma_{m,n}$ and 16: let *indexNrep* be the list of indices of the sorted entries. k = 117: while $assignment No < \delta^{min}$ do 18: n = indexNrep[k]19: Select $SU_n \in \mathcal{S}^{nrep}$ as a candidate and calculate 20: $\tau_{m,n}$ value to achieve P^d using (3). if $\tau_{m,n} \leq remainingTime[n]$ then 21: remainingTime[n] = remainingTime[n] -22: $assignmentNo = assignmentNo + 1, S^{rep} =$ 23: $\mathcal{S}^{rep} \cup \{SU_n\}, \mathcal{S}^{nrep} = \mathcal{S}^{nrep} \setminus \{SU_n\}$ end if 24: 25: k = k + 1end while 26: end if 27: 28: end for

other methods. The values for the other parameters are given in Table I.

By using (4), we obtain $\delta^{max}=10$ for the given P^f and ${}_{th}Q^f$ values. The reader should note that the presented results are for a single frame. Hence, the cumulative effect will be much higher if multiple frames are considered. Furthermore, the processing order of the channels is important for the given heuristics as they converge to local optimal solutions. Even though the channels are ordered naturally in the given pseudo-code, we also run both heuristics with randomly ordered channels 20 times. The results given below for the heuristics are the best of the 21 runs in terms of energy consumption.

We first observe the total energy consumption and its individual components in Figs. 2(a) and 2(b) for μ^{SNR} values of -5 dB and 2 dB, respectively. For low μ^{SNR} , the sensing component of the energy consumption is more dominant. On

TABLE I. I drameers values	
М	40
Ν	$\{160, 180, 200, 220, 240\}$
f_s	$\{1 \text{ kHz}, 10 \text{ kHz}\}$
μ^{SNR}	[-10 dB, 5 dB] with 1 dB increments
δ^{min}	3
P^{f}	0.01
α	[1.1, 3] with 0.1 increments
T^s	αT^s_{opt}
ϵ	10^{-6}
P^s	1000 mW
E_n^{rep}	1 mJ $\forall n$
P_{min}^d	0.5
$_{th}Q^d$	0.9
$_{th}Q^f$	0.1

TABLE I: Parameters values

the other hand, reporting energy consumption becomes the major component when μ^{SNR} is higher. As we can see, the reporting energy consumption is similar in both cases. Hence, the difference stems from the sensing energy consumption. With high μ^{SNR} , the time required to achieve a particular detection probability decreases, which in turn decreases the required sensing time. In both cases, TXT achieves the worst performance since its objective does not consider the energy consumption at all. On the other hand, the performance of EE is always superior compared to REM for low SNR because it prioritizes the sensing energy. Contrariwise, REM achieves lower total energy for high SNR value since it first considers the reporting energy component.

The effect of changing μ^{SNR} on total sensing energy consumption can be seen in Fig. 3. Fig. 3(a) shows a broader range whereas Fig. 3(b) shows the high SNR regime. Initially, increasing μ^{SNR} values have a significant impact on total energy consumption for all methods whereas beyond a certain point the benefits are minimal. In this case, EE provides 7% improvement over the next best method, namely SEM heuristic, when μ^{SNR} is -10 dB. Moreover, the improvement over other methods is much better when μ^{SNR} assumes higher values which can be seen in Fig. 3(b). As an example, using EE results in 22% reduction in total energy consumption compared to the next best method, REM this time, when μ^{SNR} is 0 dB. In addition, both figures support our previous claim that SEM achieves better performance than REM for low SNR values, while the reverse is true for high SNR values.

Fig. 4 illustrates the change in total energy consumption with respect to the increase in the number of SUs. Apart from TXT method, all schemes yield better results as N increases. The main reason for this performance improvement is the diversity brought by the added SUs. That is to say, with more SUs, the probability of finding an SU with a high $\gamma_{m,n}$ value increases for a given channel m. On the other hand, TXT method shows slight variations since its goal is not related to





Fig. 2: Energy consumption profiles with N = 200, $\delta^{min} = 3$, $\alpha = 2$.

energy consumption.

700

600

500

300

200

100

Ê 400

Energy

The total energy consumption and its individual components for $f_s = 10$ kHz case are presented in Fig. 5. In comparison to Fig. 2, increasing f_s has a similar effect as increasing SNR value. However, the effect of SNR is more prominent. For instance, with all other factors constant, increasing μ^{SNR} from -5 dB to 2 dB (almost a fivefold increase) results in nearly 83% reduction in energy consumption for EE. On the other hand, increasing f_s tenfold from 1 kHz to 10 kHz gives 76% decrease for EE. These observations are in accordance with (3). In addition, similar to the case in Fig 3(b), with a higher sampling rate, REM heuristic provides lower energy consumption than SEM.

The energy consumption values for various values of α are given in Figs. 6(a) and 6(b) for μ^{SNR} values -5 dB and 2 dB, respectively. As α is not a parameter for TXT, it is not affected by the change in α . For low α values, the results for SEM and REM are not shown because both heuristics fail to provide a feasible solution. For the low SNR regime, both EE and SEM produce lower energy consumption with increasing α but the decrease is marginal. Unlike SEM and EE, the results for REM first decrease and then start to increase. The rationale behind this pattern can be explained as follows:



Fig. 3: Effect of μ^{SNR} on total sensing energy consumption with $N = 200, \, \delta^{min} = 3, \, \alpha = 2.$



Fig. 4: Effect of number of SUs on total sensing energy consumption with μ^{SNR} = -5 dB, δ^{min} = 3, α = 2.



Fig. 5: Energy consumption profiles with N = 200, $\delta^{min} = 3$, $\alpha = 2$, fs = 10 kHz, $\mu^{SNR} = -5$ dB.

since REM prefers SUs that are already assigned a channel for sensing when selecting SUs for channel m, a long sensing duration causes SUs with low $\gamma_{m,n}$ to be assigned to channel m. We observe that sensing energy component dominates in low SNR regime, so this causes an increase in total energy consumption for REM. On the contrary, for high SNR regime REM produces lower energy consumption values as reporting energy component is the dominating factor. Both figures show that with only a small amount of additional sensing time, great energy savings are possible.

To sum up, all three energy minimization methods (EE, SEM, and REM) provide significant energy savings compared to a pure transmission time maximization technique. In all cases, EE achieves the best energy values whereas the performance of SEM and REM depend on the parameter values. On the one hand, a low μ^{SNR} or a high α favors SEM. On the other hand, a high μ^{SNR} or a high f_s supports REM. As both heuristics have very low complexities, both can be executed in a short amount of time, and one can select the method with the better energy consumption.

VII. CONCLUSION

In this paper, we have formulated the energy-efficient cooperative sensing scheduling problem for a CRN and presented various approaches for this problem. Each scheme ensures the minimum detection probability constraint as a PU protection criteria and the maximum false alarm probability constraint as CRN operability criteria in each channel. EE, SEM, and REM aim to minimize energy expenditure for sensing while TXT minimizes time spent for the sensing task in order to leave more time for data transmission. We have investigated the performance of our proposals with various parameters. To find the optimal solution we have employed the outer linearization method. Numerical evaluations have shown that by sacrificing very little data transmission time, significant amount of energy can be saved. Furthermore, reporting energy is an important factor in the energy consumption, especially, when the SNR or sampling frequency is high.

As future work, we plan to incorporate different fusion rules, e.g. AND, MAJORITY, etc. into our model. Moreover, we



Fig. 6: Effect of sensing duration (T_s) under low and high SNR values with $N = 200, \delta^{min} = 3$.

also would like to analyze the impact of channel switching delay and energy consumption of channel switching on sensing energy consumption. Another point to pursue is to treat false alarm probabilities as decision variables, and jointly optimize them together with sensing times.

APPENDIX A Proof of Theorem 1

The first derivative of $P_{m,n}^d$ with respect to $\tau_{m,n}$ is

$$\frac{dP_{m,n}^d}{d\tau_{m,n}} = \frac{\gamma_{m,n}\sqrt{f_s}}{2\sqrt{\tau_{m,n}}\sqrt{2\pi}\sqrt{2\gamma_{m,n}+1}}A_{m,n}$$

where

$$A_{m,n} = \exp\left(-\frac{1}{2}\left(\frac{Q^{-1}(P_{m,n}^{f}) - \sqrt{\tau_{m,n}f_{s}}\gamma_{m,n}}{\sqrt{2\gamma_{m,n} + 1}}\right)^{2}\right).$$

The first derivative is always positive, hence, $P_{m,n}^d$ is an increasing function of $\tau_{m,n}$.

The second derivative of $P_{m,n}^d$ with respect to $\tau_{m,n}$ is given by

$$\frac{d^2 P_{m,n}^d}{d\tau_{m,n}^2} = \frac{\gamma_{m,n}\sqrt{f_s}A_{m,n}}{4\sqrt{2\pi}\sqrt{2\gamma_{m,n}+1}} \left[-\frac{1}{\sqrt{\tau_{m,n}^3}} + \frac{\gamma_{m,n}\sqrt{f_s}}{\tau_{m,n}(2\gamma_{m,n}+1)} \right]$$
$$\left(\mathcal{Q}^{-1}(P_{m,n}^f) - \sqrt{\tau_{m,n}f_s}\gamma_{m,n}\right) \right].$$

The second derivative is negative if

$$\frac{-1}{\sqrt{\tau_{m,n}^3}} + \frac{\gamma_{m,n}\sqrt{f_s}}{\tau_{m,n}(2\gamma_{m,n}+1)} (\mathcal{Q}^{-1}(P_{m,n}^f) - \sqrt{\tau_{m,n}f_s}\gamma_{m,n}) < 0.$$

Reducing the $\tau_{m,n}$ term leads to

$$-\frac{1}{\sqrt{\tau_{m,n}}} + \frac{\gamma_{m,n}\sqrt{f_s}}{(2\gamma_{m,n}+1)} (\mathcal{Q}^{-1}(P^f_{m,n}) - \sqrt{\tau_{m,n}f_s}\gamma_{m,n}) < 0.$$

Thus, $P_{m,n}^d$ is a concave function of $\tau_{m,n}$ if the condition in (2) is satisfied.

APPENDIX B Proof of Lemma 1

By combining (1) and (2), we get

$$-\frac{1}{\sqrt{\tau_{m,n}}} + \frac{\gamma_{m,n}\sqrt{f_s}\mathcal{Q}^{-1}(P_{m,n}^d)}{\sqrt{2\gamma_{m,n}+1}} < 0.$$

The first term is always negative, whereas the second term is negative if $P_{m,n}^d > 0.5$. Since it is reasonable to assume a $P_{m,n}^d$ value greater than 0.5, we can safely say that $P_{m,n}^d$ is a concave function of $\tau_{m,n}$ most of the time.

APPENDIX C Proof of Theorem 2

Let $\tau_{\mathbf{n}}$ denote the τ vector with n entries that consists of $\tau_{m,n}$ values for channel m. Moreover, let $f_{m,k}$ and $h_{m,k}$ denote $(1 - P_{m,k}^d)$, and $f_{m,1}f_{m,2} \dots f_{m,k}$, respectively. The proof is by induction on the number of elements in \mathcal{S}_m denoted by $|\mathcal{S}_m|$.

|S_m| = 2: Without loss of generality, assume SUs 1 and 2 are in S_m. We can rewrite Q^d_m as 1 − h_{m,2}. The gradient of h_{m,2} is given by

$$\frac{\partial h_{m,2}}{\partial \tau_2} = \left[-\frac{\gamma_{m,1}\sqrt{f_s}A_{m,1}}{2\sqrt{\tau_{m,1}}\sqrt{2\pi}\sqrt{2\gamma_{m,1}+1}} f_{m,2} , -\frac{\gamma_{m,2}\sqrt{f_s}A_{m,2}}{2\sqrt{\tau_{m,2}}\sqrt{2\pi}\sqrt{2\gamma_{m,2}+1}} f_{m,1} \right]$$

where

$$A_{m,n} = \exp\left(-\frac{1}{2}\left(\frac{\mathcal{Q}^{-1}(P_{m,n}^f) - \sqrt{\tau_{m,n}f_s}\gamma_{m,n}}{\sqrt{2\gamma_{m,n}+1}}\right)^2\right).$$

Both terms are always negative, thus, $h_{m,2}$ is a decreasing function of τ_2 . Therefore, Q_m^d is an increasing function of τ_2 since $\frac{\partial Q_m^d}{\partial \tau_2} = -\frac{\partial h_{m,2}}{\partial \tau_2}$. In addition, as shown in Lemma 2, both $f_{m,1}$ and $f_{m,2}$ are non-negative, decreasing, and convex functions so their multiplication, $h_{m,2}$, is also convex [21], which leads to the concavity of Q_m^d .

• Let us assume that the theorem holds for $|S_m| = k$ and show that it also holds for $|S_m| = k + 1$. This time Q_m^d can be written as

$$Q_m^d = 1 - h_{m,k+1} = 1 - h_{m,k} f_{m,k+1}.$$

The gradient of $h_{m,k+1}$ is given by

$$\frac{\partial h_{m,k+1}}{\partial \tau_{\mathbf{k+1}}} = \left[\frac{\partial h_{m,k}}{\partial \tau_{\mathbf{k+1}}} f_{m,k+1}, h_{m,k} \frac{\partial f_{m,k+1}}{\partial \tau_{\mathbf{k+1}}}\right]$$

Let us focus on the first term. Since $\frac{\partial h_{m,k}}{\partial \tau_{k+1}}$ is negative by induction, and $f_{m,k+1}$ is non-negative, then their multiplication is negative. For the second term, $h_{m,k}$ is a non-negative function, and $\frac{\partial f_{m(k+1)}}{\partial \tau_{k+1}}$ is negative by Lemma 2. Thus, their multiplication is also negative. Since both terms are negative, $h_{m,k+1}$ is a decreasing function of τ_{k+1} .

We apply the same logic as in the previous step to prove the convexity of $h_{m,k+1}$. Both $h_{m,k}$ and $f_{m,k+1}$ are decreasing convex functions (convexity of $h_{m,k}$ comes from induction), then their multiplication, $h_{m,k+1}$, is also convex. Thus, Q_m^d is a concave and increasing function of τ_{k+1} .

Proving both the base step and the induction step leads to the conclusion that Q_m^d is an increasing concave function of $\tau_{m,n}$ if (2) is satisfied.

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References

- Y. Liang, Y. Zeng, E. Peh, and A. Hoang, "Sensingthroughput tradeoff for cognitive radio networks," *IEEE Transactions on Wireless Communications*, vol. 7, no. 4, pp. 1326–1337, 2008.
- [2] G. Ganesan and Y. Li, "Cooperative spectrum sensing in cognitive radio networks," in *First IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks*, 2005, pp. 137–143.
- [3] C. Song and Q. Zhang, "Cooperative spectrum sensing with multi-channel coordination in cognitive radio networks," in *IEEE International Conference on Communications (ICC)*, 2010, pp. 1–5.
- [4] Y. Liu, S. Xie, Y. Zhang, R. Yu, and V. C. M. Leung, "Energy-efficient spectrum discovery for cognitive radio green networks," *Mobile Networks and Applications*, March 2011.
- [5] H. Su and X. Zhang, "Opportunistic energy-aware channel sensing schemes for dynamic spectrum access networks," in *IEEE Global Telecommunications Conference* (*GLOBECOM*), 2010, pp. 1–5.
- [6] —, "Power-efficient periodic spectrum sensing for cognitive MAC in dynamic spectrum access networks," in *IEEE Wireless Communications and Networking Conference (WCNC)*, 2010, pp. 1–6.

- [8] Y. Wu and D. Tsang, "Energy-efficient spectrum sensing and transmission for cognitive radio system," *IEEE Communications Letters*, vol. 15, no. 5, pp. 545–547, 2011.
- [9] Y. Pei, Y. Liang, K. Teh, and K. Li, "Energy-efficient design of sequential channel sensing in cognitive radio networks: Optimal sensing strategy, power allocation, and sensing order," *IEEE Journal on Selected Areas in Communications*, vol. 29, no. 8, pp. 1648–1659, 2011.
- [10] S. Huang, H. Chen, Y. Zhang, and F. Zhao, "Energyefficient cooperative spectrum sensing with amplifyand-forward relaying," *IEEE Communications Letters*, vol. 16, no. 4, 2012.
- [11] R. Deng, J. Chen, C. Yuen, P. Cheng, and Y. Sun, "Energy-efficient cooperative spectrum sensing by optimal scheduling in sensor-aided cognitive radio networks," *IEEE Transactions on Vehicular Technology*, vol. 61, no. 2, pp. 716–725, 2012.
- [12] S. Maleki, S. Chepuri, and G. Leus, "Energy and throughput efficient strategies for cooperative spectrum sensing in cognitive radios," in *IEEE 12th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, 2011.
- [13] C. Lee and W. Wolf, "Energy efficient techniques for cooperative spectrum sensing in cognitive radios," in 5th IEEE Consumer Communications and Networking Conference (CCNC), 2008, pp. 968–972.
- [14] E. C. Peh, Y.-C. Liang, Y. L. Guan, and Y. Pei, "Energyefficient cooperative spectrum sensing in cognitive radio networks," in *IEEE Global Telecommunications Conference (GLOBECOM)*, 2011.
- [15] X. Sun, T. Zhang, and D. Tsang, "Optimal energyefficient cooperative sensing scheduling for cognitive radio networks with QoS guarantee," in 7th IEEE International Wireless Communications and Mobile Computing Conference (IWCMC), 2011, pp. 1825–1830.
- [16] T. Zhang and D. Tsang, "Optimal cooperative sensing scheduling for energy-efficient cognitive radio networks," in *IEEE International Conference on Computer Communications (INFOCOM)*, 2011, pp. 2723–2731.
- [17] X. Hao, M. Cheung, V. Wong, and V. Leung, "A coalition formation game for energy-efficient cooperative spectrum sensing in cognitive radio networks with multiple channels," in *IEEE Global Telecommunications Conference* (*GLOBECOM 2011*), 2011, pp. 1–6.
- [18] M. Alouini and A. Goldsmith, "Capacity of Rayleigh fading channels under different adaptive transmission and diversity-combining techniques," *IEEE Transactions on Vehicular Technology*, vol. 48, no. 4, pp. 1165–1181, 1999.
- [19] A. Conejo, E. Castillo, R. Minguez, and R. Garcia-Bertrand, *Decomposition Techniques in Mathematical Programming*. Springer, 2006.
- [20] C. A. Floudas, Nonlinear and Mixed Integer Optimization. Fundamentals and Applications. Oxford University Press, New York, 1995.

[21] M. M. Kantrowitz, Robert Neumann, "Optimization for products of concave functions," *Rendiconti del Circolo Matematico di Palermo*, vol. 54-2, pp. 291–302, 2005.



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